Capacitance of Transmission Line:

The capacitance between the conductors is the charge per unit of potential difference.

$$C = \frac{q}{V} \qquad F/m$$

- Capacitance between parallel conductors is a constant depending on the size and spacing of the conductors.
- For the length of transmission line less than 80 Km, the effect of capacitance is usually neglected.
- For longer lines the capacitance becomes increasingly important and has to be accounted for.
- The line capacitance draws a leading sinusoidal current called the charging current which is drawn even when the line is open circuited at the far end.
- Charging current effects the voltage drop along the line, efficiency, power factor and stability of the power system.

✓ **Gauss theorem states that at any instant of time, the total electric flux through any closed surface (A) is equal to the total charge enclosed by that surface.**

$$D = \frac{q}{2\pi x} \qquad coulomb/m^2$$

Where
q **- Charge on the conductor coulomb /m**
x **- distance, from the conductor to the point where the electric flux density is computed.**
D **- The electric flux density.**

$$\text{The electric field intensity} = \frac{\text{The electric flux density}}{\text{The permittivity of the medium}}$$

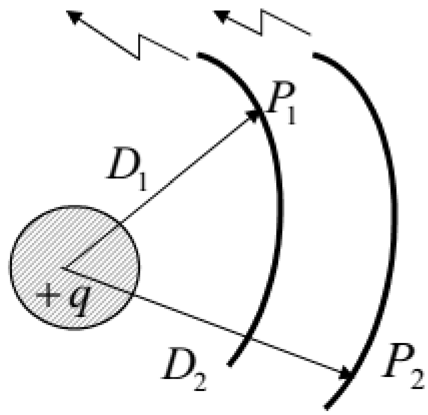
$$E = \frac{D}{\varepsilon} = \frac{q}{2\pi x \varepsilon} \qquad V/m$$

Where ε - actual permittivity of material .

$$\varepsilon_r = \varepsilon / \varepsilon_0$$

ε_r - relative permittivity ; ε_0 - permittivity of free space

where ε_0 is the permittivity of free space and is equal to 8.85×10^{-12} F/m.



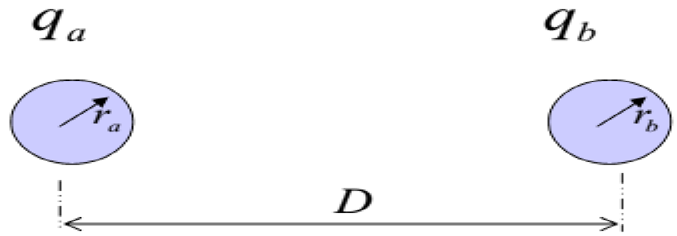
The potential difference between cylinders from position D_1 to D_2 is defined as the work done in moving a unit charge of one coulomb from D_2 to D_1 through the electric field produced by the charge on the conductor. This is given by

$$v_{12} = \int_{D_1}^{D_2} E \, dx$$

$$= \int_{D_1}^{D_2} \frac{q}{2\pi x \varepsilon} dx = \frac{q}{2\pi \varepsilon} \ln \frac{D_2}{D_1} \quad volts$$

❖ **This is the instantaneous voltage drop between *P*1 and *P*2.**

Capacitance of a two wire line :



From equation.

$$v_{12} = \frac{q}{2\pi \varepsilon} \ln \frac{D_2}{D_1} \quad volts$$

$$v_{ab} \ due \ \ to \ q_a = \frac{q_a}{2\pi\varepsilon} \ln \frac{D}{r_a} \quad volt$$

And voltage due to q_b , calculated by :

$$v_{ba} = \frac{q_b}{2\pi\varepsilon} \ln \frac{D}{r_b} \quad volt$$

$$v_{ab} = -v_{ba}$$

$$\therefore \quad v_{ab} = -\frac{q_b}{2\pi\varepsilon} \ln \frac{D}{r_b} = \frac{q_b}{2\pi\varepsilon} \ln \left(\frac{D}{r_b} \right)^{-1}$$

$$\therefore \quad v_{ab} \ due \ \ to \ q_b = \frac{q_b}{2\pi\varepsilon} \ln \frac{r_b}{D} \quad volt$$

- By the principle of superposition the voltage drop from conductor (a) to conductor (b) due to charges on both conductors is the sum of the voltage drop caused by each charge alone.

$$\boldsymbol{V_{ab} = V_{ab(qa)} + V_{ab(qb)}}$$

$$\therefore \quad v_{ab} = \frac{1}{2\pi\varepsilon} \left(q_a \ln \frac{D}{r_a} + q_b \ln \frac{r_b}{D} \right) volt$$

For two-wire line $q_a = -\ q_b$, so that :

$$v_{ab} = \frac{q_a}{2\pi\varepsilon} \ln \frac{D^2}{r_a r_b} = \frac{q_a}{2\pi\varepsilon} \ln \left(\frac{D}{\sqrt{r_a r_b}} \right)^2$$

$$V_{xy} = \frac{1}{2\pi\varepsilon} \left[q \ln \frac{D_{yx}}{D_{xx}} - q \ln \frac{D_{yy}}{D_{xy}} \right]$$
$$= \frac{q}{2\pi\varepsilon} \ln \frac{D_{yx} D_{xy}}{D_{xx} D_{yy}}$$

$$= \frac{q_a}{\pi\varepsilon} \ln\left(\frac{D}{\sqrt{r_a r_b}}\right) \quad volts$$

$$C_{ab} = \frac{q_a}{v_{ab}} = \frac{\pi\varepsilon}{\ln\left(\frac{D}{\sqrt{r_a r_b}}\right)} \quad Farads \quad / meter$$

If $r_a = r_b = r$ (radius of two conductors are same)

$$\therefore \quad C_{ab} = \frac{\pi\varepsilon}{\ln\left(\frac{D}{r}\right)} \quad F / m$$

Coulomb's constant	$\kappa = 1/4\pi\epsilon_0$

$$\boldsymbol{\varepsilon o = \frac{1}{4\pi k}}$$

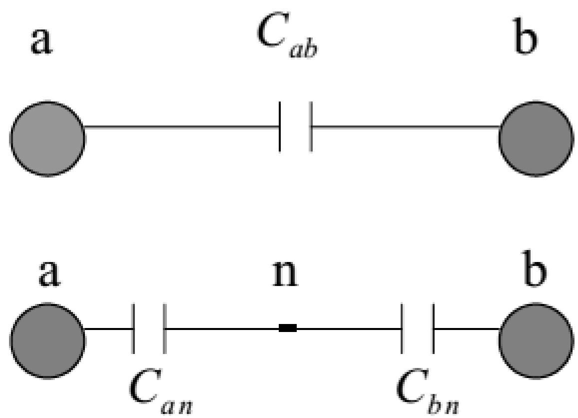
$$If, \varepsilon_r = 1, and\ \varepsilon_0 = \frac{1}{4\pi \times 9 \times 10^9} \quad F/m$$

$$\therefore \quad C_{ab} = \frac{10^{-9}}{36 \ln\left(\frac{D}{r}\right)} \quad F / m$$

$$= \frac{1}{36 \ln \frac{D}{r}} \quad \mu F / Km$$

- The line-to-line capacitance is equivalently considered as two equal capacitances is sense.

- The voltage across the lines divides equally between the capacitances such that the neutral point **n** is at the ground potential.



Cab - capacitance between conductors (a) and (b) .
Ca n - capacitance between conductor (a) and neutral .
Cb n - capacitance between conductor (b) and neutral .

$$C_{an} = C_{bn} = C_n = 2\,C_{ab}$$

$$\therefore \quad C_n = 2 \times \frac{1}{36 \ln \frac{D}{r}} = \frac{1}{18 \ln \frac{D}{r}}$$

$$\boxed{\therefore \quad C_n = \frac{0.0555}{\ln \frac{D}{r}} \qquad \mu F / Km}$$

The associated line charging current is

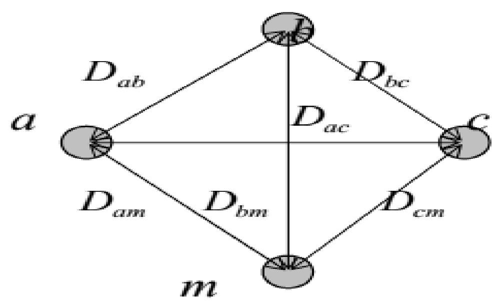
$$I_C = j\omega\, C_{12} V_{12} \quad \text{A/km}$$

- Capacitive reactance between one conductor and neutral is :

$$X_c = \frac{1}{2\pi f C_n}$$

<u>Potential difference between two conductors of a group of charged conductors</u>

- The voltage drop between the two conductors is the sum of the voltage drops due to each charged conductor.
- The voltage drop between (a) to (b):



$$V_{ij} = \frac{1}{2\pi\varepsilon_0}\sum_{k=1}^{n} q_k \ln\frac{D_{kj}}{D_{ki}}$$

When $k = i$, D_{ii} is the distance between the surface of the conductor and its center, namely its radius r.

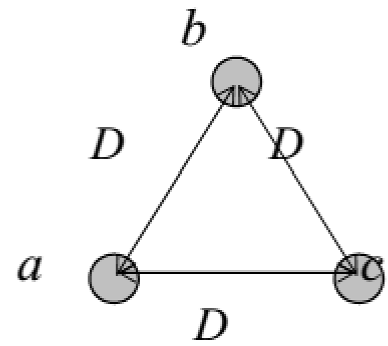
$$v_{ab} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D_{ab}}{r_a} + q_b \ln\frac{r_b}{D_{ba}} + q_c \ln\frac{D_{cb}}{D_{ca}} + \ldots \ldots\ldots + q_m \ln\frac{D_{mb}}{D_{ma}} \right) volts$$

In similar manner :

$$v_{ac} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D_{ac}}{r_a} + q_b \ln\frac{D_{bc}}{D_{ba}} + q_c \ln\frac{r_c}{D_{ca}} + \ldots \ldots\ldots + q_m \ln\frac{D_{mc}}{D_{ma}} \right) volts$$

Capacitance of three – phase line:

a- Equilateral spacing :



By using the equation

$$v_{ab} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D_{ab}}{r_a} + q_b \ln\frac{r_b}{D_{ba}} + q_c \ln\frac{D_{cb}}{D_{ca}} + \ldots \ldots\ldots + q_m \ln\frac{D_{mb}}{D_{ma}} \right) volts$$

$$v_{ab} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D} + q_c \underbrace{\ln\frac{D}{D}}_{0} \right)$$

$$v_{ac} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D}{r} + q_b \ln\frac{D}{D} + q_c \ln\frac{r}{D} \right)$$

$$v_{ab} + v_{ac} = \frac{1}{2\pi\varepsilon}\left(2q_a \ln\frac{D}{r} + (q_b + q_c)\ln\frac{r}{D} \right) \quad Volts$$

But $q_b + q_c = -q_a$

$$\therefore\; v_{ab} + v_{ac} = \frac{1}{2\pi\varepsilon}\left(2q_a \ln\frac{D}{r} - q_a \ln\frac{r}{D} \right)$$

y ln[a/b] = ln [a/b] ^ y

$$= \frac{1}{2\pi\varepsilon}\left(q_a \ln\left(\frac{D}{r}\right)^2 - q_a \ln\frac{r}{D} \right) = \frac{1}{2\pi\varepsilon}\left(q_a \ln\left(\frac{D}{r}\right)^3 \right)$$

$$= \frac{3q_a}{2\pi\varepsilon}\ln\frac{D}{r} \quad volts$$

Phasor diagram of voltages v_{ab}, v_{bc} *and* v_{ca} :

$$\vec{v}_{ab} = |v_{ab}|\angle 30^\circ$$

$$\frac{|v_{ab}|}{2} = V_{an}\cos 30^\circ = \frac{\sqrt{3}}{2}V_{an} \quad \therefore \quad |v_{ab}| = \sqrt{3}\,V_{an}$$

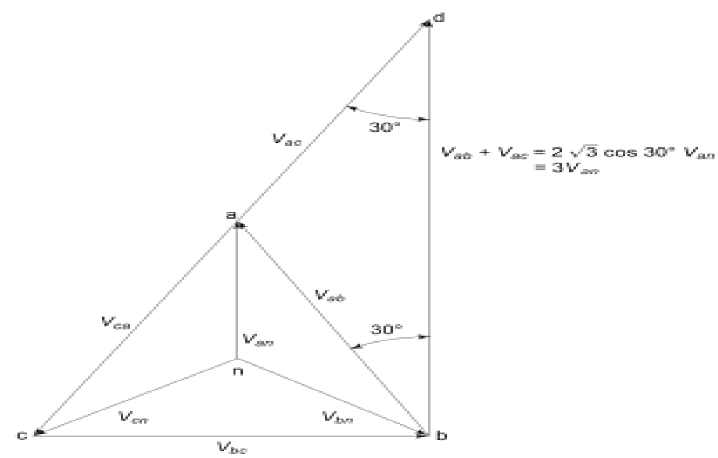
$$\vec{v}_{ab} = \sqrt{3}V_{an}\angle 30^\circ$$

$$\vec{v}_{ac} = -\vec{v}_{ca} = \sqrt{3}V_{an}\angle -30^\circ$$

$$\therefore \quad \vec{v}_{ab} + \vec{v}_{ac} = \sqrt{3}V_{an}\angle 30^\circ + \sqrt{3}V_{an}\angle -30^\circ$$

$$= \sqrt{3}V_{an} \times 2 \times \cos 30^\circ = \sqrt{3}V_{an} \times \sqrt{3}$$

$$= 3V_{an}$$



$$\therefore \quad 3V_{an} = \frac{3q_a}{2\pi\varepsilon}\ln\frac{D}{r}$$

$$V_{an} = \frac{q_a}{2\pi\varepsilon}\ln\frac{D}{r} \quad volt$$

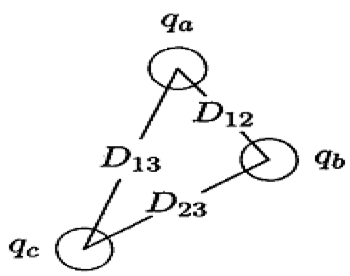
$$C_n = \frac{q_a}{V_{an}} = \frac{2\pi\varepsilon}{\ln\dfrac{D}{r}} \quad F/m \quad \text{to neutral}$$

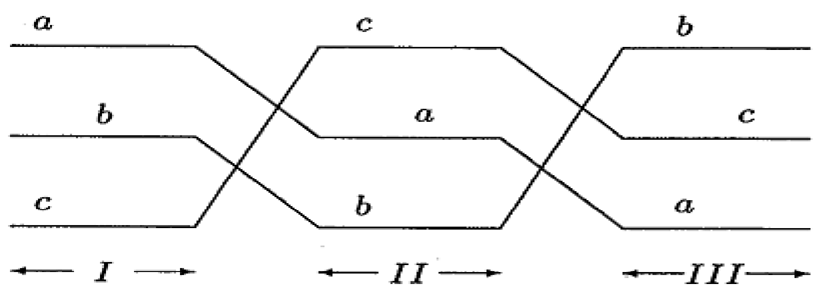
$$For \;\; \varepsilon_r = 1$$

$$\therefore \quad C_n = \frac{1}{18 \ln \frac{D}{r}} = \frac{0.0555}{\ln \frac{D}{r}} \qquad \mu F / Km$$

as in the case of single-phase lines.

b) *Unsymmetrical spacing but transposed*:





$$v_{ab1} = \frac{1}{2\pi\varepsilon}\left[q_a \ln \frac{D_{12}}{r} + q_b \ln \frac{r}{D_{12}} + q_c \ln \frac{D_{23}}{D_{31}} \right]$$

$$v_{ab2} = \frac{1}{2\pi\varepsilon}\left[q_a \ln \frac{D_{23}}{r} + q_b \ln \frac{r}{D_{23}} + q_c \ln \frac{D_{31}}{D_{12}} \right]$$

$$v_{ab3} = \frac{1}{2\pi\varepsilon}\left[q_a \ln \frac{D_{31}}{r} + q_b \ln \frac{r}{D_{31}} + q_c \ln \frac{D_{12}}{D_{23}} \right]$$

$$v_{ab} = \frac{v_{ab1} + v_{ab2} + v_{ab3}}{3}$$

$$v_{ab} = \frac{1}{6\pi\varepsilon}\left[q_a \ln \frac{D_{12} D_{23} D_{31}}{r^3} + q_b \ln \frac{r^3}{D_{12} D_{23} D_{31}} + + q_c \ln \cancel{\frac{D_{12} D_{23} D_{31}}{D_{12} D_{23} D_{31}}} \right]$$

$$v_{ab} = \frac{1}{2\pi\varepsilon}\left[q_a \ln \frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{r} + q_b \ln \frac{r}{\sqrt[3]{D_{12}D_{23}D_{31}}} \right]$$

If, $D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$ OR ==➔ $GMD = \sqrt[3]{D_{12}D_{23}D_{13}}$

$$\therefore \quad v_{ab} = \frac{1}{2\pi\varepsilon}\left[q_a \ln \frac{D_{eq}}{r} + q_b \ln \frac{r}{D_{eq}} \right]$$

➔OR $V_{ab} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln \frac{GMD}{r} + q_b \ln \frac{r}{GMD} \right)$

Similarly, $v_{ac} = \frac{1}{2\pi\varepsilon}\left[q_a \ln \frac{D_{eq}}{r} + q_c \ln \frac{r}{D_{eq}} \right]$ ➔OR $V_{ac} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln \frac{GMD}{r} + q_c \ln \frac{r}{GMD} \right)$

As in section (a):

$$v_{ab} + v_{ac} = 3V_{an} \quad and \quad q_b + q_c = -q_a$$

$$\therefore \quad v_{an} = \frac{1}{3\times 2\pi\varepsilon}\left[2\times q_a \ln \frac{D_{eq}}{r} - q_a \ln \frac{r}{D_{eq}} \right]$$

$$v_{an} = \frac{1}{3\times 2\pi\varepsilon}\left[q_a \ln \frac{D_{eq}^2}{r^2} - q_a \ln \frac{r}{D_{eq}} \right]$$

$$v_{an} = \frac{1}{3\times 2\pi\varepsilon}\left[q_a \ln \frac{D_{eq}^3}{r^3} \right]$$

$$v_{an} = \frac{q_a}{2\pi\varepsilon} \ln \frac{D_{eq}}{r} \quad volts$$

$$C_n = \frac{q_a}{v_{an}} = \frac{2\pi\varepsilon}{\ln\frac{D_{eq}}{r}} \quad F/m$$

➔OR

$$C = \frac{q_a}{V_{an}} = \frac{2\pi\varepsilon_0}{\ln\frac{GMD}{r}} \quad \text{F/m}$$

$$\therefore \quad C_n = \frac{1}{18\times\ln\frac{D_{eq}}{r}} = \frac{0.0555}{\ln\frac{D_{eq}}{r}} \quad \mu F/Km$$

➔OR

$$C = \frac{0.0556}{\ln\frac{GMD}{r}} \quad \mu\text{F/km}$$

This is of the same form as the expression for the capacitance of one phase of a single-phase line. *GMD* (geometric mean distance) is the equivalent conductor spacing. For the above three-phase line this is the cube root of the product of the three-phase spacings.